OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754(A)

Applications of Advanced Mathematics (C4)

Section A

Thursday **16 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this section is 72.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 4 printed pages.

[Turn over

Section A (36 marks)

1 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence find the range of the function $f(\theta)$, where

$$f(\theta) = 7 + 3\cos\theta + 4\sin\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

Write down the greatest possible value of $\dfrac{1}{7 + 3\cos\theta + 4\sin\theta}$. [6]

2 Find the first 4 terms in the binomial expansion of $\sqrt{4+2x}$. State the range of values of x for which the expansion is valid. [6]

3 Solve the equation

$$\sec^2\theta = 4, \quad 0 \leqslant \theta \leqslant \pi,$$

giving your answers in terms of π. [4]

4 Fig. 4 shows a sketch of the region enclosed by the curve $\sqrt{1+e^{-2x}}$, the x-axis, the y-axis and the line $x = 1$.

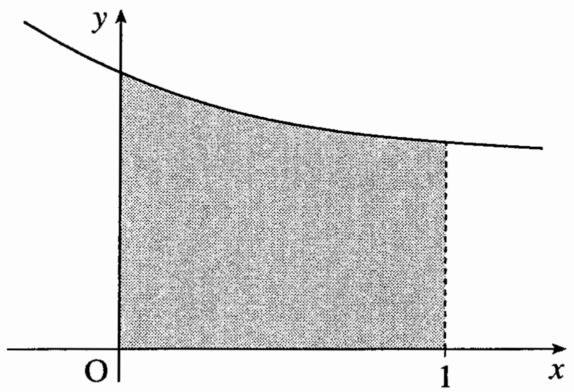

Fig. 4

Find the volume of the solid generated when this region is rotated through 360° about the x-axis. Give your answer in an exact form. [5]

5 Solve the equation $2\cos 2x = 1 + \cos x$, for $0° \leqslant x < 360°$. [7]

6 A curve has cartesian equation $y^2 - x^2 = 4$.

(i) Verify that

$$x = t - \frac{1}{t}, \quad y = t + \frac{1}{t},$$

are parametric equations of the curve. [2]

(ii) Show that $\frac{dy}{dx} = \frac{(t-1)(t+1)}{t^2+1}$. Hence find the coordinates of the stationary points of the curve. [6]

Section B (36 marks)

7 In a chemical process, the mass M grams of a chemical at time t minutes is modelled by the differential equation

$$\frac{dM}{dt} = \frac{M}{t(1+t^2)}.$$

(i) Find $\int \frac{t}{1+t^2} dt$. [3]

(ii) Find constants A, B and C such that

$$\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}.$$ [5]

(iii) Use integration, together with your results in parts **(i)** and **(ii)**, to show that

$$M = \frac{Kt}{\sqrt{1+t^2}},$$

where K is a constant. [6]

(iv) When $t = 1, M = 25$. Calculate K.

What is the mass of the chemical in the long term? [4]

[Turn over

8 A computer-controlled machine can be programmed to make cuts by entering the equation of the plane of the cut, and to drill holes by entering the equation of the line of the hole.

A 20 cm × 30 cm × 30 cm cuboid is to be cut and drilled. The cuboid is positioned relative to x-, y- and z-axes as shown in Fig. 8.1.

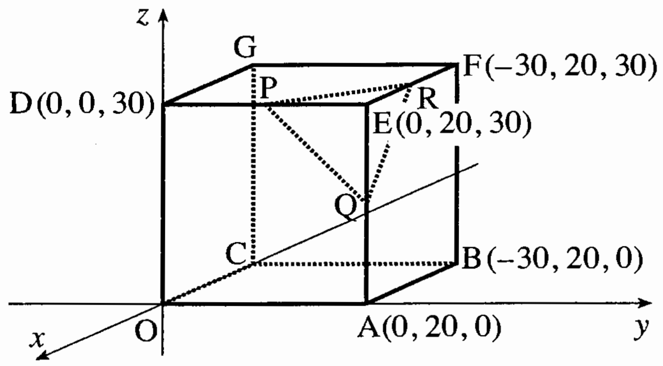

Fig. 8.1

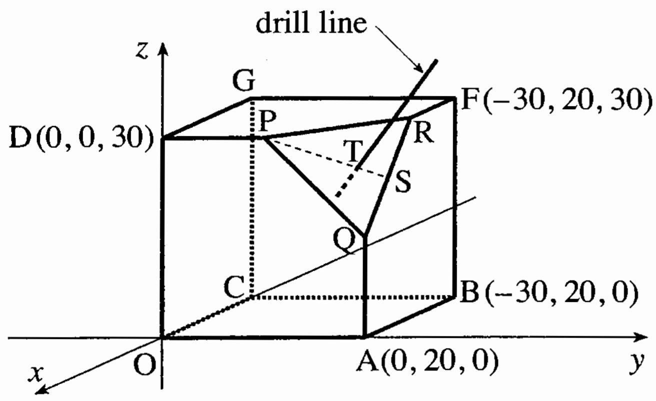

Fig. 8.2

First, a plane cut is made to remove the corner at E. The cut goes through the points P, Q and R, which are the midpoints of the sides ED, EA and EF respectively.

(i) Write down the coordinates of P, Q and R.

Hence show that $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ and $\overrightarrow{PR} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$. [4]

(ii) Show that the vector $\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$ is perpendicular to the plane through P, Q and R.

Hence find the cartesian equation of this plane. [5]

A hole is then drilled perpendicular to triangle PQR, as shown in Fig. 8.2. The hole passes through the triangle at the point T which divides the line PS in the ratio 2:1, where S is the midpoint of QR.

(iii) Write down the coordinates of S, and show that the point T has coordinates $(-5, 16\frac{2}{3}, 25)$. [4]

(iv) Write down a vector equation of the line of the drill hole.

Hence determine whether or not this line passes through C. [5]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754(A)

Applications of Advanced Mathematics (C4)

Paper A

Monday **23 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.

NOTE

- This paper will be followed by **Paper B: Comprehension.**

This question paper consists of 4 printed pages.

Section A (36 marks)

1 Solve the equation $\dfrac{2x}{x-2} - \dfrac{4x}{x+1} = 3$. [5]

2 A curve is defined parametrically by the equations

$$x = t - \ln t, \qquad y = t + \ln t \qquad (t > 0).$$

Find the gradient of the curve at the point where $t = 2$. [5]

3 A triangle ABC has vertices $A(-2, 4, 1)$, $B(2, 3, 4)$ and $C(4, 8, 3)$. By calculating a suitable scalar product, show that angle ABC is a right angle. Hence calculate the area of the triangle. [6]

4 Solve the equation $2\sin 2\theta + \cos 2\theta = 1$, for $0° \leqslant \theta < 360°$. [6]

5 **(i)** Find the cartesian equation of the plane through the point $(2, -1, 4)$ with normal vector

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$ [3]

(ii) Find the coordinates of the point of intersection of this plane and the straight line with equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 12 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}.$$ [4]

6 **(i)** Find the first three non-zero terms of the binomial expansion of $\dfrac{1}{\sqrt{4-x^2}}$ for $|x| < 2$. [4]

(ii) Use this result to find an approximation for $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$, rounding your answer to 4 significant figures. [2]

(iii) Given that $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx = \arcsin\left(\tfrac{1}{2}x\right) + c$, evaluate $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$, rounding your answer to 4 significant figures. [1]

Section B (36 marks)

7 In a game of rugby, a kick is to be taken from a point P (see Fig. 7). P is a perpendicular distance y metres from the line TOA. Other distances and angles are as shown.

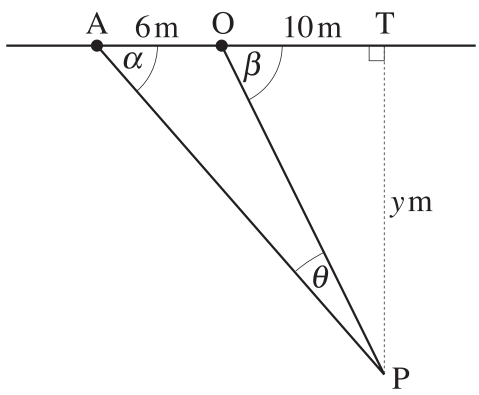

Fig. 7

(i) Show that $\theta = \beta - \alpha$, and hence that $\tan\theta = \dfrac{6y}{160 + y^2}$.

Calculate the angle θ when $y = 6$. [8]

(ii) By differentiating implicitly, show that $\dfrac{d\theta}{dy} = \dfrac{6(160 - y^2)}{(160 + y^2)^2}\cos^2\theta$. [5]

(iii) Use this result to find the value of y that maximises the angle θ. Calculate this maximum value of θ. [You need not verify that this value is indeed a maximum.] [4]

[Question 8 is printed overleaf.]

8 Some years ago an island was populated by red squirrels and there were no grey squirrels. Then grey squirrels were introduced.

The population x, in thousands, of red squirrels is modelled by the equation

$$x = \frac{a}{1+kt},$$

where t is the time in years, and a and k are constants. When $t = 0, x = 2.5$.

(i) Show that $\frac{dx}{dt} = -\frac{kx^2}{a}$. [3]

(ii) Given that the initial population of 2.5 thousand red squirrels reduces to 1.6 thousand after one year, calculate a and k. [3]

(iii) What is the long-term population of red squirrels predicted by this model? [1]

The population y, in thousands, of grey squirrels is modelled by the differential equation

$$\frac{dy}{dt} = 2y - y^2.$$

When $t = 0, y = 1$.

(iv) Express $\frac{1}{2y - y^2}$ in partial fractions. [4]

(v) Hence show by integration that $\ln\left(\frac{y}{2-y}\right) = 2t$.

Show that $y = \frac{2}{1+e^{-2t}}$. [7]

(vi) What is the long-term population of grey squirrels predicted by this model? [1]

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RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4754(A)

Applications of Advanced Mathematics (C4)

Paper A

Monday **12 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

NOTE

- This paper will be followed by **Paper B: Comprehension.**

This question paper consists of 5 printed pages and 3 blank pages.

Section A (36 marks)

1 Fig. 1 shows part of the graph of $y = \sin x - \sqrt{3}\cos x$.

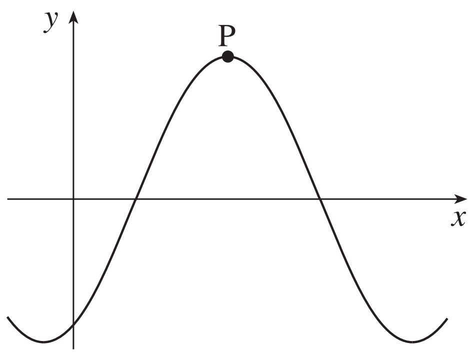

Fig. 1

Express $\sin x - \sqrt{3}\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0 \leqslant \alpha \leqslant \frac{1}{2}\pi$.

Hence write down the exact coordinates of the turning point P. [6]

2 **(i)** Given that

$$\frac{3+2x^2}{(1+x)^2(1-4x)} = \frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{1-4x},$$

where A, B and C are constants, find B and C, and show that $A = 0$. [4]

(ii) Given that x is sufficiently small, find the first three terms of the binomial expansions of $(1+x)^{-2}$ and $(1-4x)^{-1}$.

Hence find the first three terms of the expansion of $\dfrac{3+2x^2}{(1+x)^2(1-4x)}$. [4]

3 Given that $\sin(\theta+\alpha) = 2\sin\theta$, show that $\tan\theta = \dfrac{\sin\alpha}{2-\cos\alpha}$.

Hence solve the equation $\sin(\theta + 40°) = 2\sin\theta$, for $0° \leqslant \theta \leqslant 360°$. [7]

4 **(a)** The number of bacteria in a colony is increasing at a rate that is proportional to the square root of the number of bacteria present. Form a differential equation relating x, the number of bacteria, to the time t. [2]

(b) In another colony, the number of bacteria, y, after time t minutes is modelled by the differential equation

$$\frac{dy}{dt} = \frac{10000}{\sqrt{y}}.$$

Find y in terms of t, given that $y = 900$ when $t = 0$. Hence find the number of bacteria after 10 minutes. [6]

5 **(i)** Show that $\int x e^{-2x}\,dx = -\frac{1}{4}e^{-2x}(1+2x) + c$. [3]

A vase is made in the shape of the volume of revolution of the curve $y = x^{\frac{1}{2}}e^{-x}$ about the x-axis between $x = 0$ and $x = 2$ (see Fig. 5).

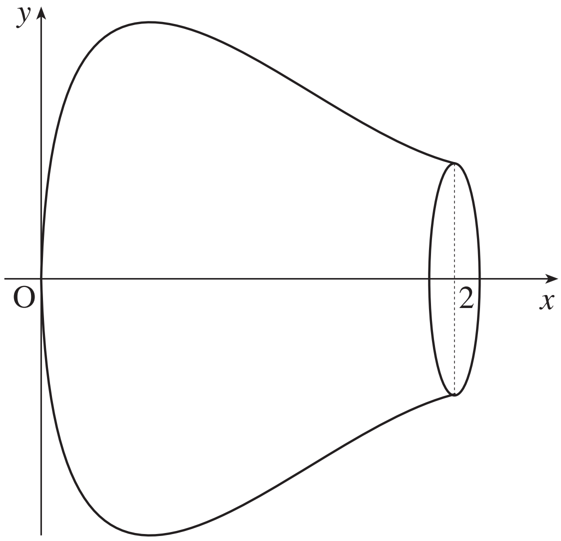

Fig. 5

(ii) Show that this volume of revolution is $\frac{1}{4}\pi\left(1 - \frac{5}{e^4}\right)$. [4]

[Turn over

Section B (36 marks)

6 Fig. 6 shows the arch ABCD of a bridge.

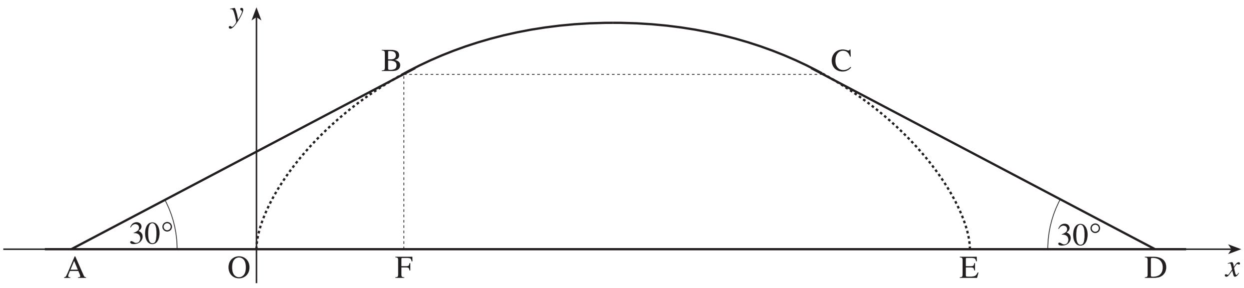

Fig. 6

The section from B to C is part of the curve OBCE with parametric equations

$$x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta) \text{ for } 0 \leqslant \theta \leqslant 2\pi,$$

where a is a constant.

(i) Find, in terms of a,

(A) the length of the straight line OE,

(B) the maximum height of the arch. [4]

(ii) Find $\dfrac{dy}{dx}$ in terms of θ. [3]

The straight line sections AB and CD are inclined at 30° to the horizontal, and are tangents to the curve at B and C respectively. BC is parallel to the x-axis. BF is parallel to the y-axis.

(iii) Show that at the point B the parameter θ satisfies the equation

$$\sin\theta = \frac{1}{\sqrt{3}}(1 - \cos\theta).$$

Verify that $\theta = \frac{2}{3}\pi$ is a solution of this equation.

Hence show that BF $= \frac{3}{2}a$, and find OF in terms of a, giving your answer exactly. [6]

(iv) Find BC and AF in terms of a.

Given that the straight line distance AD is 20 metres, calculate the value of a. [5]

7

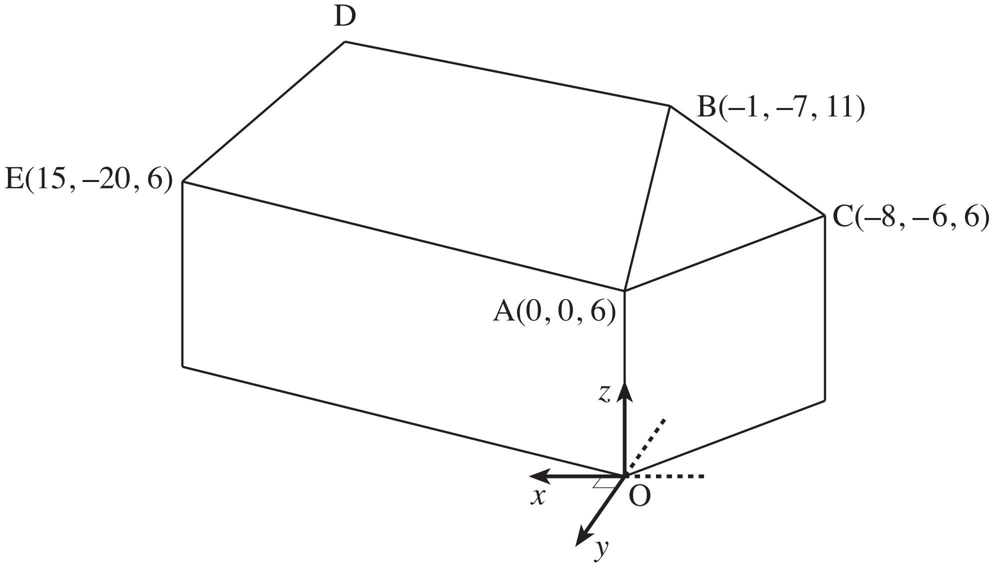

Fig. 7

Fig. 7 illustrates a house. All units are in metres. The coordinates of A, B, C and E are as shown. BD is horizontal and parallel to AE.

(i) Find the length AE. [2]

(ii) Find a vector equation of the line BD. Given that the length of BD is 15 metres, find the coordinates of D. [4]

(iii) Verify that the equation of the plane ABC is

$$-3x + 4y + 5z = 30.$$

Write down a vector normal to this plane. [4]

(iv) Show that the vector $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to the plane ABDE. Hence find the equation of the plane ABDE. [4]

(v) Find the angle between the planes ABC and ABDE. [4]

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4754(A)/01

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4)

Paper A

TUESDAY 23 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
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NOTE

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **6** printed pages and **2** blank pages.

  [Turn over

Section A (36 marks)

1 Solve the equation $\dfrac{1}{x} + \dfrac{x}{x+2} = 1.$ [4]

2 Fig. 2 shows part of the curve $y = \sqrt{1 + x^3}$.

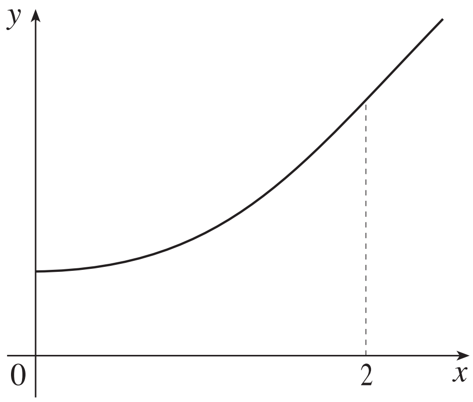

Fig. 2

(i) Use the trapezium rule with 4 strips to estimate $\displaystyle\int_0^2 \sqrt{1+x^3}\,\mathrm{d}x$, giving your answer correct to 3 significant figures. [3]

(ii) Chris and Dave each estimate the value of this integral using the trapezium rule with 8 strips. Chris gets a result of 3.25, and Dave gets 3.30. One of these results is correct. Without performing the calculation, state with a reason which is correct. [2]

3 **(i)** Use the formula for $\sin(\theta + \phi)$, with $\theta = 45°$ and $\phi = 60°$, to show that $\sin 105° = \frac{\sqrt{3}+1}{2\sqrt{2}}$. [4]

(ii) In triangle ABC, angle BAC = 45°, angle ACB = 30° and AB = 1 unit (see Fig. 3).

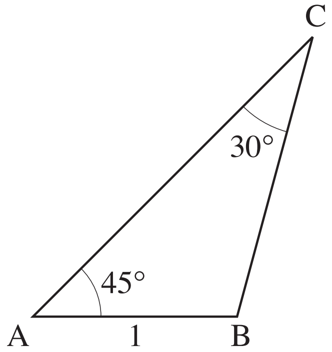

Fig. 3

Using the sine rule, together with the result in part (**i**), show that $\mathrm{AC} = \frac{\sqrt{3}+1}{\sqrt{2}}$. [3]

4 Show that $\frac{1+\tan^2\theta}{1-\tan^2\theta} = \sec 2\theta$.

Hence, or otherwise, solve the equation $\frac{1+\tan^2\theta}{1-\tan^2\theta} = 2$, for $0° \leqslant \theta \leqslant 180°$. [7]

5 Find the first four terms in the binomial expansion of $(1+3x)^{\frac{1}{3}}$.

State the range of values of x for which the expansion is valid. [5]

6 **(i)** Express $\frac{1}{(2x+1)(x+1)}$ in partial fractions. [3]

(ii) A curve passes through the point $(0, 2)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{(2x+1)(x+1)}.$$

Show by integration that $y = \frac{4x+2}{x+1}$. [5]

Section B (36 marks)

7 Fig. 7 shows the curve with parametric equations

$$x = \cos\theta,\ y = \sin\theta - \tfrac{1}{8}\sin 2\theta,\ 0 \leqslant \theta < 2\pi.$$

The curve crosses the x-axis at points $\mathrm{A}(1, 0)$ and $\mathrm{B}(-1, 0)$, and the positive y-axis at C. D is the maximum point of the curve, and E is the minimum point.

The solid of revolution formed when this curve is rotated through 360° about the x-axis is used to model the shape of an egg.

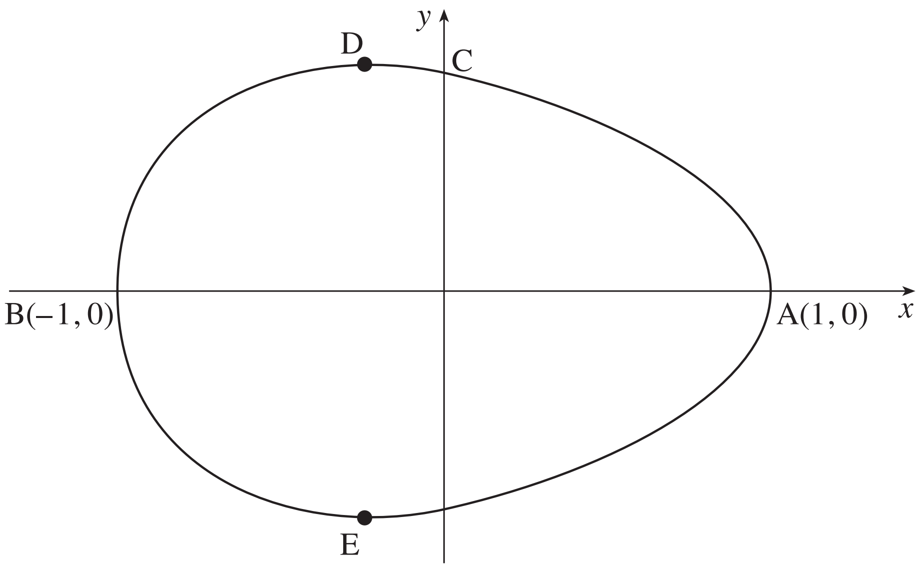

Fig. 7

(i) Show that, at the point A, $\theta = 0$. Write down the value of θ at the point B, and find the coordinates of C. [4]

(ii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ.

Hence show that, at the point D,

$$2\cos^2\theta - 4\cos\theta - 1 = 0.$$ [5]

(iii) Solve this equation, and hence find the y-coordinate of D, giving your answer correct to 2 decimal places. [5]

The cartesian equation of the curve (for $0 \leqslant \theta \leqslant \pi$) is

$$y = \tfrac{1}{4}(4 - x)\sqrt{1 - x^2}.$$

(iv) Show that the volume of the solid of revolution of this curve about the x-axis is given by

$$\tfrac{1}{16}\pi \int_{-1}^{1} \left(16 - 8x - 15x^2 + 8x^3 - x^4\right)\mathrm{d}x.$$

Evaluate this integral. [6]

8 A pipeline is to be drilled under a river (see Fig. 8). With respect to axes Oxyz, with the x-axis pointing East, the y-axis North and the z-axis vertical, the pipeline is to consist of a straight section AB from the point A$(0, -40, 0)$ to the point B$(40, 0, -20)$ directly under the river, and another straight section BC. All lengths are in metres.

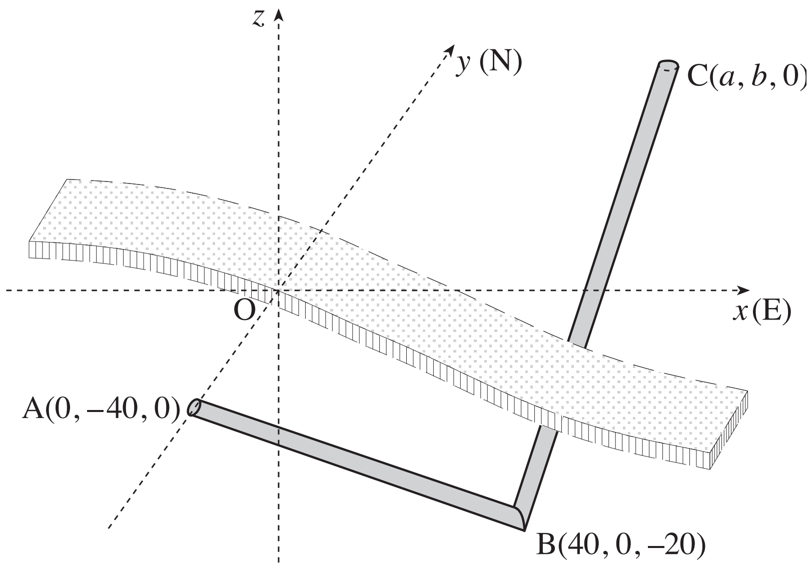

Fig. 8

(i) Calculate the distance AB. [2]

The section BC is to be drilled in the direction of the vector $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$.

(ii) Find the angle ABC between the sections AB and BC. [4]

The section BC reaches ground level at the point C$(a, b, 0)$.

(iii) Write down a vector equation of the line BC. Hence find a and b. [5]

(iv) Show that the vector $6\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ is perpendicular to the plane ABC. Hence find the cartesian equation of this plane. [5]

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4)

Paper A

4754(A)/01

THURSDAY 14 JUNE 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

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NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants to be determined, and $0° < \alpha < 90°$.

Hence solve the equation $\sin\theta - 3\cos\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [7]

2 Write down normal vectors to the planes $2x + 3y + 4z = 10$ and $x - 2y + z = 5$.

Hence show that these planes are perpendicular to each other. [4]

3 Fig. 3 shows the curve $y = \ln x$ and part of the line $y = 2$.

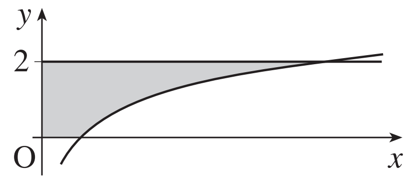

Fig. 3

The shaded region is rotated through 360° about the y-axis.

(i) Show that the volume of the solid of revolution formed is given by $\displaystyle\int_0^2 \pi e^{2y}\,dy$. [3]

(ii) Evaluate this, leaving your answer in an exact form. [3]

4 A curve is defined by parametric equations

$$x = \frac{1}{t} - 1, \quad y = \frac{2+t}{1+t}.$$

Show that the cartesian equation of the curve is $y = \dfrac{3+2x}{2+x}$. [4]

5 Verify that the point $(-1, 6, 5)$ lies on both the lines

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}.$$

Find the acute angle between the lines. [7]

6 Two students are trying to evaluate the integral $\int_1^2 \sqrt{1+\mathrm{e}^{-x}}\,\mathrm{d}x$.

Sarah uses the trapezium rule with 2 strips, and starts by constructing the following table.

x	1	1.5	2
$\sqrt{1+\mathrm{e}^{-x}}$	1.1696	1.1060	1.0655

(i) Complete the calculation, giving your answer to 3 significant figures. [2]

Anish uses a binomial approximation for $\sqrt{1+\mathrm{e}^{-x}}$ and then integrates this.

(ii) Show that, provided e^{-x} is suitably small, $\left(1+\mathrm{e}^{-x}\right)^{\frac{1}{2}} \approx 1+\frac{1}{2}\mathrm{e}^{-x}-\frac{1}{8}\mathrm{e}^{-2x}$. [3]

(iii) Use this result to evaluate $\int_1^2 \sqrt{1+\mathrm{e}^{-x}}\,\mathrm{d}x$ approximately, giving your answer to 3 significant figures. [3]

Section B (36 marks)

7 Data suggest that the number of cases of infection from a particular disease tends to oscillate between two values over a period of approximately 6 months.

(a) Suppose that the number of cases, P thousand, after time t months is modelled by the equation $P = \dfrac{2}{2 - \sin t}$. Thus, when $t = 0, P = 1$.

(i) By considering the greatest and least values of $\sin t$, write down the greatest and least values of P predicted by this model. [2]

(ii) Verify that P satisfies the differential equation $\dfrac{dP}{dt} = \frac{1}{2}P^2 \cos t$. [5]

(b) An alternative model is proposed, with differential equation

$$\frac{dP}{dt} = \tfrac{1}{2}(2P^2 - P)\cos t. \qquad (*)$$

As before, $P = 1$ when $t = 0$.

(i) Express $\dfrac{1}{P(2P-1)}$ in partial fractions. [4]

(ii) Solve the differential equation (*) to show that

$$\ln\left(\frac{2P-1}{P}\right) = \tfrac{1}{2}\sin t. \qquad [5]$$

This equation can be rearranged to give $P = \dfrac{1}{2 - e^{\frac{1}{2}\sin t}}$.

(iii) Find the greatest and least values of P predicted by this model. [4]

8

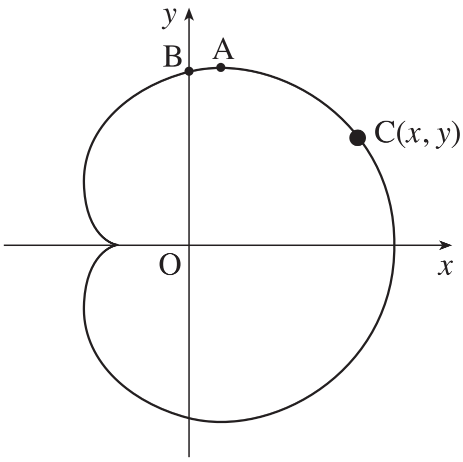

Fig. 8

In a theme park ride, a capsule C moves in a vertical plane (see Fig. 8). With respect to the axes shown, the path of C is modelled by the parametric equations

$$x = 10\cos\theta + 5\cos 2\theta, \quad y = 10\sin\theta + 5\sin 2\theta, \qquad (0 \leqslant \theta < 2\pi),$$

where x and y are in metres.

(i) Show that $\dfrac{dy}{dx} = -\dfrac{\cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta}$.

Verify that $\dfrac{dy}{dx} = 0$ when $\theta = \frac{1}{3}\pi$. Hence find the exact coordinates of the highest point A on the path of C. [6]

(ii) Express $x^2 + y^2$ in terms of θ. Hence show that

$$x^2 + y^2 = 125 + 100\cos\theta. \qquad [4]$$

(iii) Using this result, or otherwise, find the greatest and least distances of C from O. [2]

You are given that, at the point B on the path vertically above O,

$$2\cos^2\theta + 2\cos\theta - 1 = 0.$$

(iv) Using this result, and the result in part (**ii**), find the distance OB. Give your answer to 3 significant figures. [4]

RECOGNISING ACHIEVEMENT

ADVANCED GCE **4754/01A**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

TUESDAY 22 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **4** printed pages.

Section A (36 marks)

1 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $3\cos\theta + 4\sin\theta = 2$ for $-\pi \leqslant \theta \leqslant \pi$. [7]

2 **(i)** Find the first three terms in the binomial expansion of $\dfrac{1}{\sqrt{1-2x}}$. State the set of values of x for which the expansion is valid. [5]

(ii) Hence find the first three terms in the series expansion of $\dfrac{1+2x}{\sqrt{1-2x}}$. [3]

3 Fig. 3 shows part of the curve $y = 1 + x^2$, together with the line $y = 2$.

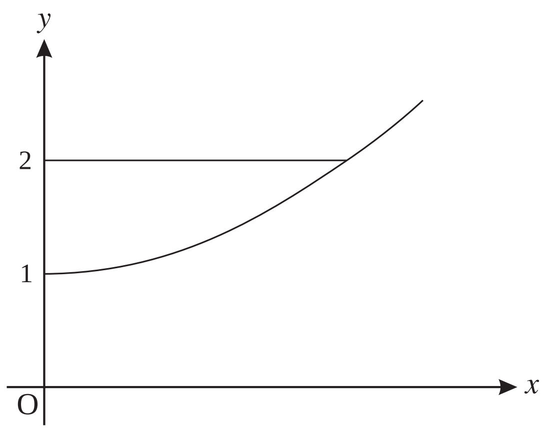

Fig. 3

The region enclosed by the curve, the y-axis and the line $y = 2$ is rotated through $360°$ about the y-axis. Find the volume of the solid generated, giving your answer in terms of π. [5]

4 The angle θ satisfies the equation $\sin(\theta + 45°) = \cos\theta$.

(i) Using the exact values of $\sin 45°$ and $\cos 45°$, show that $\tan\theta = \sqrt{2} - 1$. [5]

(ii) Find the values of θ for $0° < \theta < 360°$. [2]

5 Express $\dfrac{4}{x(x^2+4)}$ in partial fractions. [6]

6 Solve the equation $\operatorname{cosec}\theta = 3$, for $0° < \theta < 360°$. [3]

Section B (36 marks)

7 A glass ornament OABCDEFG is a truncated pyramid on a rectangular base (see Fig. 7). All dimensions are in centimetres.

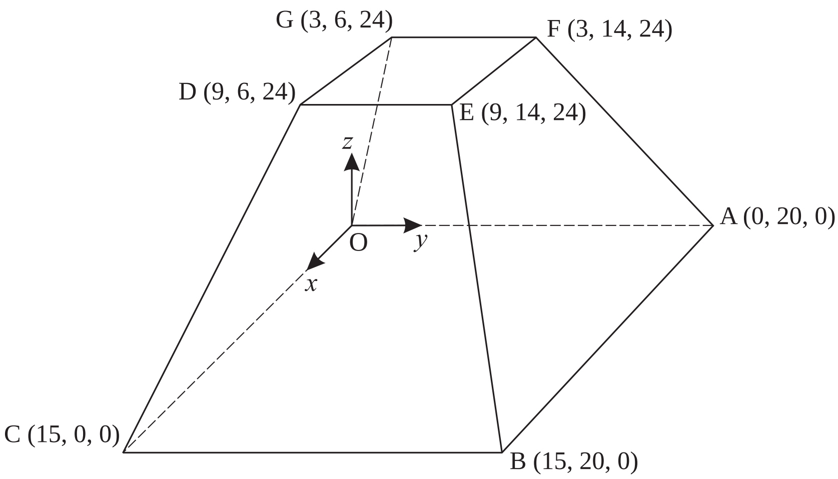

Fig. 7

(i) Write down the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. [2]

(ii) Find the length of the edge CD. [2]

(iii) Show that the vector $4\mathbf{i} + \mathbf{k}$ is perpendicular to the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. Hence find the cartesian equation of the plane BCDE. [5]

(iv) Write down vector equations for the lines OG and AF.

Show that they meet at the point P with coordinates $(5, 10, 40)$. [5]

You may assume that the lines CD and BE also meet at the point P.

The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ height.

(v) Find the volumes of the pyramids POABC and PDEFG.

Hence find the volume of the ornament. [4]

8 A curve has equation

$$x^2 + 4y^2 = k^2,$$

where k is a positive constant.

(i) Verify that

$$x = k\cos\theta, \qquad y = \tfrac{1}{2}k\sin\theta,$$

are parametric equations for the curve. [3]

(ii) Hence or otherwise show that $\dfrac{dy}{dx} = -\dfrac{x}{4y}$. [3]

(iii) Fig. 8 illustrates the curve for a particular value of k. Write down this value of k. [1]

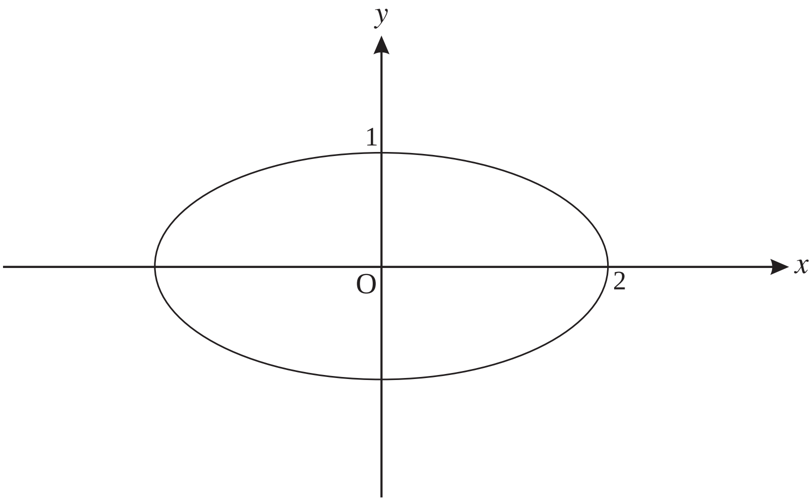

Fig. 8

(iv) Copy Fig. 8 and on the same axes sketch the curves for $k = 1$, $k = 3$ and $k = 4$. [3]

On a map, the curves represent the contours of a mountain. A stream flows down the mountain. Its path on the map is always at right angles to the contour it is crossing.

(v) Explain why the path of the stream is modelled by the differential equation

$$\frac{dy}{dx} = \frac{4y}{x}.$$ [2]

(vi) Solve this differential equation.

Given that the path of the stream passes through the point $(2, 1)$, show that its equation is $y = \dfrac{x^4}{16}$. [6]

ADVANCED GCE

MATHEMATICS (MEI)

4754/01A

Applications of Advanced Mathematics (C4) Paper A

WEDNESDAY 21 MAY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
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- Answer **all** the questions.
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- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **4** printed pages.

Section A (36 marks)

1 Express $\dfrac{x}{x^2-4}+\dfrac{2}{x+2}$ as a single fraction, simplifying your answer. [3]

2 Fig. 2 shows the curve $y=\sqrt{1+e^{2x}}$.

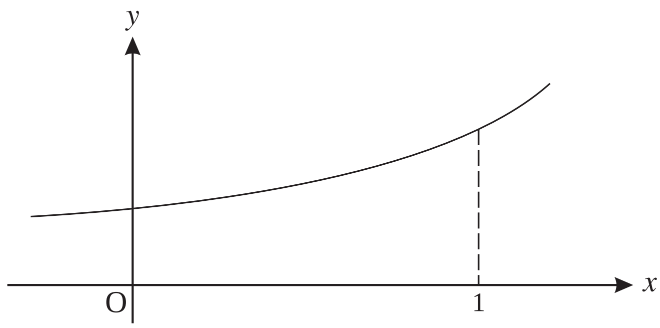

Fig. 2

The region bounded by the curve, the x-axis, the y-axis and the line $x=1$ is rotated through $360°$ about the x-axis.

Show that the volume of the solid of revolution produced is $\frac{1}{2}\pi(1+e^2)$. [4]

3 Solve the equation $\cos 2\theta=\sin\theta$ for $0\leqslant\theta\leqslant 2\pi$, giving your answers in terms of π. [7]

4 Given that $x=2\sec\theta$ and $y=3\tan\theta$, show that $\dfrac{x^2}{4}-\dfrac{y^2}{9}=1$. [3]

5 A curve has parametric equations $x=1+u^2$, $y=2u^3$.

(i) Find $\dfrac{dy}{dx}$ in terms of u. [3]

(ii) Hence find the gradient of the curve at the point with coordinates $(5,\ 16)$. [2]

6 **(i)** Find the first three non-zero terms of the binomial series expansion of $\dfrac{1}{\sqrt{1+4x^2}}$, and state the set of values of x for which the expansion is valid. [5]

(ii) Hence find the first three non-zero terms of the series expansion of $\dfrac{1-x^2}{\sqrt{1+4x^2}}$. [3]

7 Express $\sqrt{3}\sin x-\cos x$ in the form $R\sin(x-\alpha)$, where $R>0$ and $0<\alpha<\frac{1}{2}\pi$. Express α in the form $k\pi$.

Find the exact coordinates of the maximum point of the curve $y=\sqrt{3}\sin x-\cos x$ for which $0<x<2\pi$. [6]

Section B (36 marks)

8 The upper and lower surfaces of a coal seam are modelled as planes ABC and DEF, as shown in Fig. 8. All dimensions are metres.

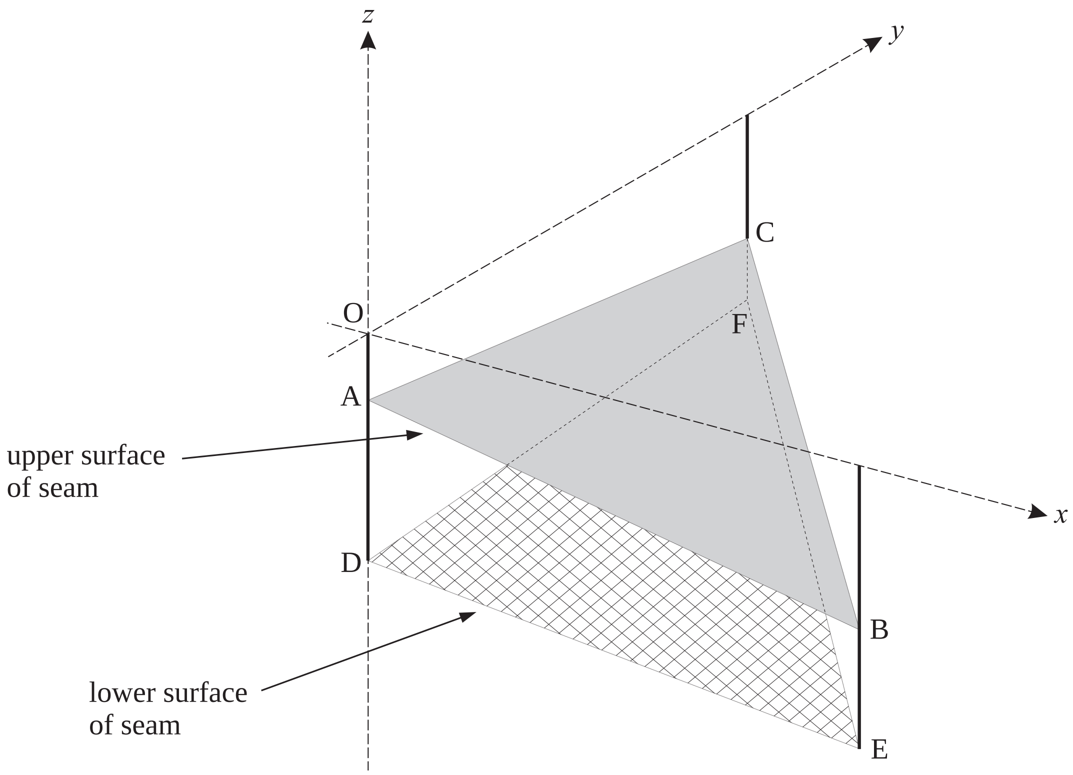

Fig. 8

Relative to axes Ox (due east), Oy (due north) and Oz (vertically upwards), the coordinates of the points are as follows.

A: $(0,\ 0,\ -15)$ B: $(100,\ 0,\ -30)$ C: $(0,\ 100,\ -25)$

D: $(0,\ 0,\ -40)$ E: $(100,\ 0,\ -50)$ F: $(0,\ 100,\ -35)$

(i) Verify that the cartesian equation of the plane ABC is $3x + 2y + 20z + 300 = 0$. [3]

(ii) Find the vectors $\overrightarrow{DE}$ and $\overrightarrow{DF}$. Show that the vector $2\mathbf{i} - \mathbf{j} + 20\mathbf{k}$ is perpendicular to each of these vectors. Hence find the cartesian equation of the plane DEF. [6]

(iii) By calculating the angle between their normal vectors, find the angle between the planes ABC and DEF. [4]

It is decided to drill down to the seam from a point R $(15,\ 34,\ 0)$ in a line perpendicular to the upper surface of the seam. This line meets the plane ABC at the point S.

(iv) Write down a vector equation of the line RS.

Calculate the coordinates of S. [5]

9 A skydiver drops from a helicopter. Before she opens her parachute, her speed $v\,\mathrm{m\,s^{-1}}$ after time t seconds is modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10\mathrm{e}^{-\frac{1}{2}t}.$$

When $t = 0$, $v = 0$.

(i) Find v in terms of t. [4]

(ii) According to this model, what is the speed of the skydiver in the long term? [2]

She opens her parachute when her speed is $10\,\mathrm{m\,s^{-1}}$. Her speed t seconds after this is $w\,\mathrm{m\,s^{-1}}$, and is modelled by the differential equation

$$\frac{\mathrm{d}w}{\mathrm{d}t} = -\tfrac{1}{2}(w-4)(w+5).$$

(iii) Express $\dfrac{1}{(w-4)(w+5)}$ in partial fractions. [4]

(iv) Using this result, show that $\dfrac{w-4}{w+5} = 0.4\mathrm{e}^{-4.5t}$. [6]

(v) According to this model, what is the speed of the skydiver in the long term? [2]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4754A**

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Tuesday 13 January 2009
Morning

Duration: 1 hour 30 minutes

* 4 7 5 4 A *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
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- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

2R–8H16

Section A (36 marks)

1 Express $\dfrac{3x+2}{x(x^2+1)}$ in partial fractions. **[6]**

2 Show that $(1+2x)^{\frac{1}{3}} = 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \ldots$, and find the next term in the expansion.

State the set of values of x for which the expansion is valid. **[6]**

3 Vectors $\mathbf{a}$ and $\mathbf{b}$ are given by $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Find constants λ and μ such that $\lambda\mathbf{a} + \mu\mathbf{b} = 4\mathbf{j} - 3\mathbf{k}$. **[5]**

4 Prove that $\cot\beta - \cot\alpha = \dfrac{\sin(\alpha - \beta)}{\sin\alpha\sin\beta}$. **[3]**

5 **(i)** Write down normal vectors to the planes $2x - y + z = 2$ and $x - z = 1$.

Hence find the acute angle between the planes. **[4]**

(ii) Write down a vector equation of the line through $(2, 0, 1)$ perpendicular to the plane $2x - y + z = 2$. Find the point of intersection of this line with the plane. **[4]**

6 **(i)** Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute, expressing α in terms of π. **[4]**

(ii) Write down the derivative of $\tan\theta$.

Hence show that $\displaystyle\int_0^{\frac{1}{3}\pi} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}\,d\theta = \frac{\sqrt{3}}{4}$. **[4]**

Section B (36 marks)

7 Scientists can estimate the time elapsed since an animal died by measuring its body temperature.

(i) Assuming the temperature goes down at a constant rate of 1.5 degrees Fahrenheit per hour, estimate how long it will take for the temperature to drop

(A) from 98 °F to 89 °F,

(B) from 98 °F to 80 °F. **[2]**

In practice, rate of temperature loss is not likely to be constant. A better model is provided by Newton's law of cooling, which states that the temperature θ in degrees Fahrenheit t hours after death is given by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - \theta_0),$$

where θ_0 °F is the air temperature and k is a constant.

(ii) Show by integration that the solution of this equation is $\theta = \theta_0 + Ae^{-kt}$, where A is a constant. **[5]**

The value of θ_0 is 50, and the initial value of θ is 98. The initial rate of temperature loss is 1.5 °F per hour.

(iii) Find A, and show that $k = 0.031\,25$. **[4]**

(iv) Use this model to calculate how long it will take for the temperature to drop

(A) from 98 °F to 89 °F,

(B) from 98 °F to 80 °F. **[5]**

(v) Comment on the results obtained in parts **(i)** and **(iv)**. **[1]**

[Question 8 is printed overleaf.]

8 Fig. 8 illustrates a hot air balloon on its side. The balloon is modelled by the volume of revolution about the x-axis of the curve with parametric equations

$$x = 2 + 2\sin\theta, \quad y = 2\cos\theta + \sin 2\theta, \quad (0 \leqslant \theta \leqslant 2\pi).$$

The curve crosses the x-axis at the point A (4, 0). B and C are maximum and minimum points on the curve. Units on the axes are metres.

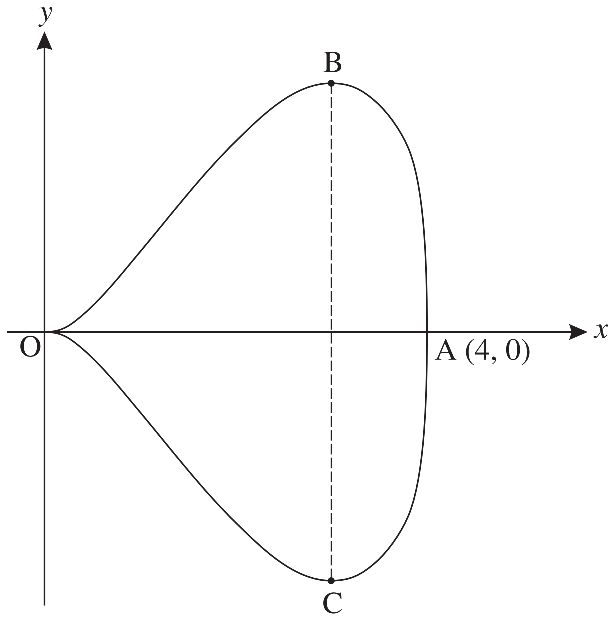

Fig. 8

(i) Find $\dfrac{dy}{dx}$ in terms of θ. **[4]**

(ii) Verify that $\dfrac{dy}{dx} = 0$ when $\theta = \frac{1}{6}\pi$, and find the exact coordinates of B.

Hence find the maximum width BC of the balloon. **[5]**

(iii) (*A*) Show that $y = x\cos\theta$.

(*B*) Find $\sin\theta$ in terms of x and show that $\cos^2\theta = x - \frac{1}{4}x^2$.

(*C*) Hence show that the cartesian equation of the curve is $y^2 = x^3 - \frac{1}{4}x^4$. **[7]**

(iv) Find the volume of the balloon. **[3]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 1 June 2009
Morning

Duration: 1 hour 30 minutes

* 4 7 5 4 A *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
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- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
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INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $4\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $4\cos\theta - \sin\theta = 3$, for $0 \leqslant \theta \leqslant 2\pi$. **[7]**

2 Using partial fractions, find $\displaystyle\int \frac{x}{(x+1)(2x+1)}\,dx$. **[7]**

3 A curve satisfies the differential equation $\dfrac{dy}{dx} = 3x^2 y$, and passes through the point $(1, 1)$. Find y in terms of x. **[4]**

4 The part of the curve $y = 4 - x^2$ that is above the x-axis is rotated about the y-axis. This is shown in Fig. 4.

Find the volume of revolution produced, giving your answer in terms of π. **[5]**

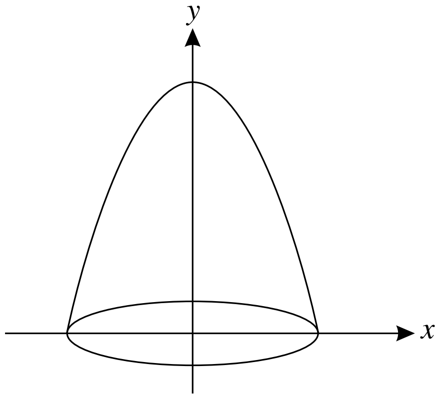

Fig. 4

5 A curve has parametric equations

$$x = at^3, \quad y = \frac{a}{1+t^2},$$

where a is a constant.

Show that $\dfrac{dy}{dx} = \dfrac{-2}{3t(1+t^2)^2}$.

Hence find the gradient of the curve at the point $(a, \frac{1}{2}a)$. **[7]**

6 Given that $\operatorname{cosec}^2\theta - \cot\theta = 3$, show that $\cot^2\theta - \cot\theta - 2 = 0$.

Hence solve the equation $\operatorname{cosec}^2\theta - \cot\theta = 3$ for $0° \leqslant \theta \leqslant 180°$. **[6]**

Section B (36 marks)

7 When a light ray passes from air to glass, it is deflected through an angle. The light ray ABC starts at point A (1, 2, 2), and enters a glass object at point B (0, 0, 2). The surface of the glass object is a plane with normal vector **n**. Fig. 7 shows a cross-section of the glass object in the plane of the light ray and **n**.

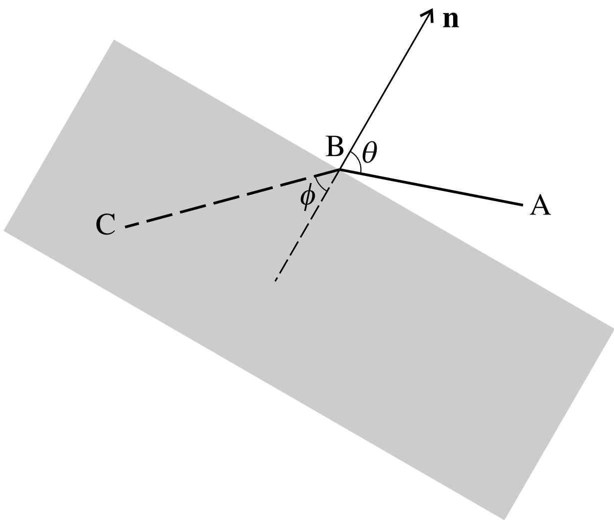

Fig. 7

(i) Find the vector $\overrightarrow{AB}$ and a vector equation of the line AB. **[2]**

The surface of the glass object is a plane with equation $x + z = 2$. AB makes an acute angle θ with the normal to this plane.

(ii) Write down the normal vector **n**, and hence calculate θ, giving your answer in degrees. **[5]**

The line BC has vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$. This line makes an acute angle ϕ with the normal to the plane.

(iii) Show that $\phi = 45°$. **[3]**

(iv) Snell's Law states that $\sin\theta = k \sin\phi$, where k is a constant called the refractive index. Find k. **[2]**

The light ray leaves the glass object through a plane with equation $x + z = -1$. Units are centimetres.

(v) Find the point of intersection of the line BC with the plane $x + z = -1$. Hence find the distance the light ray travels through the glass object. **[5]**

[Question 8 is printed overleaf.]

OCR
RECOGNISING ACHIEVEMENT

8 Archimedes, about 2200 years ago, used regular polygons inside and outside circles to obtain approximations for π.

(i) Fig. 8.1 shows a regular 12-sided polygon inscribed in a circle of radius 1 unit, centre O. AB is one of the sides of the polygon. C is the midpoint of AB. Archimedes used the fact that the circumference of the circle is greater than the perimeter of this polygon.

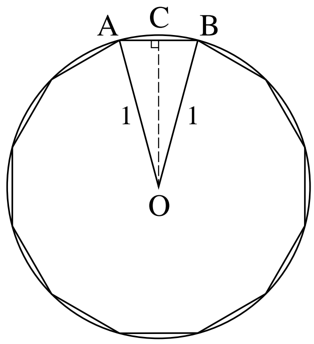

Fig. 8.1

(A) Show that AB = $2\sin 15°$. **[2]**

(B) Use a double angle formula to express $\cos 30°$ in terms of $\sin 15°$. Using the exact value of $\cos 30°$, show that $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$. **[4]**

(C) Use this result to find an exact expression for the perimeter of the polygon.

Hence show that $\pi > 6\sqrt{2-\sqrt{3}}$. **[2]**

(ii) In Fig. 8.2, a regular 12-sided polygon lies outside the circle of radius 1 unit, which touches each side of the polygon. F is the midpoint of DE. Archimedes used the fact that the circumference of the circle is less than the perimeter of this polygon.

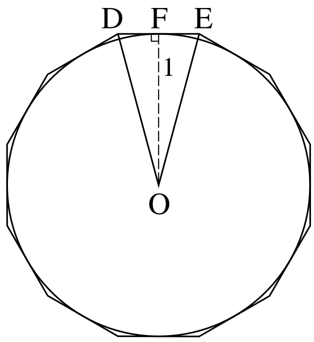

Fig. 8.2

(A) Show that DE = $2\tan 15°$. **[2]**

(B) Let $t = \tan 15°$. Use a double angle formula to express $\tan 30°$ in terms of t.

Hence show that $t^2 + 2\sqrt{3}t - 1 = 0$. **[3]**

(C) Solve this equation, and hence show that $\pi < 12\left(2-\sqrt{3}\right)$. **[4]**

(iii) Use the results in parts **(i)**(C) and **(ii)**(C) to establish upper and lower bounds for the value of π, giving your answers in decimal form. **[2]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4754A**

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 15 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Find the first three terms in the binomial expansion of $\dfrac{1+2x}{(1-2x)^2}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[7]**

2 Show that $\cot 2\theta = \dfrac{1-\tan^2\theta}{2\tan\theta}$.

Hence solve the equation

$$\cot 2\theta = 1 + \tan\theta \quad \text{for } 0^\circ < \theta < 360^\circ.$$ **[7]**

3 A curve has parametric equations

$$x = e^{2t}, \quad y = \frac{2t}{1+t}.$$

(i) Find the gradient of the curve at the point where $t = 0$. **[6]**

(ii) Find y in terms of x. **[2]**

4 The points A, B and C have coordinates $(1, 3, -2)$, $(-1, 2, -3)$ and $(0, -8, 1)$ respectively.

(i) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **[2]**

(ii) Show that the vector $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ is perpendicular to the plane ABC. Hence find the equation of the plane ABC. **[5]**

5 **(i)** Verify that the lines $\mathbf{r} = \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ meet at the point $(1, 3, 2)$. **[3]**

(ii) Find the acute angle between the lines. **[4]**

Section B (36 marks)

6 In Fig. 6, OAB is a thin bent rod, with OA = a metres, AB = b metres and angle OAB = 120°. The bent rod lies in a vertical plane. OA makes an angle θ above the horizontal. The vertical height BD of B above O is h metres. The horizontal through A meets BD at C and the vertical through A meets OD at E.

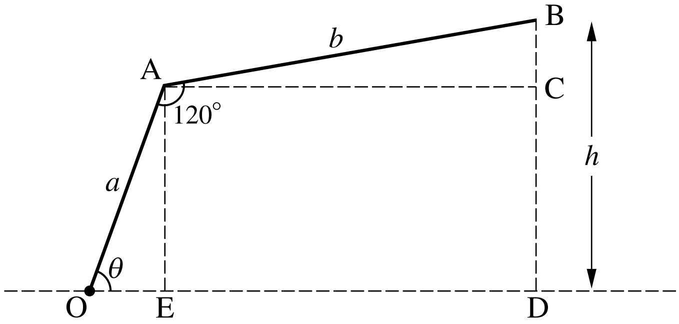

Fig. 6

(i) Find angle BAC in terms of θ. Hence show that

$$h = a\sin\theta + b\sin(\theta - 60°).$$ **[3]**

(ii) Hence show that $h = (a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta$. **[3]**

The rod now rotates about O, so that θ varies. You may assume that the formulae for h in parts **(i)** and **(ii)** remain valid.

(iii) Show that OB is horizontal when $\tan\theta = \dfrac{\sqrt{3}b}{2a + b}$. **[3]**

In the case when $a = 1$ and $b = 2$, $h = 2\sin\theta - \sqrt{3}\cos\theta$.

(iv) Express $2\sin\theta - \sqrt{3}\cos\theta$ in the form $R\sin(\theta - \alpha)$. Hence, for this case, write down the maximum value of h and the corresponding value of θ. **[7]**

[Question 7 is printed overleaf.]

7 Fig. 7 illustrates the growth of a population with time. The proportion of the ultimate (long term) population is denoted by x, and the time in years by t. When $t = 0$, $x = 0.5$, and as t increases, x approaches 1.

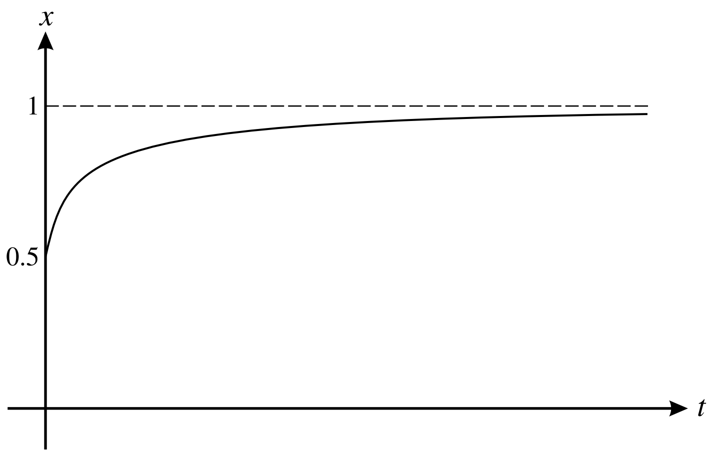

Fig. 7

One model for this situation is given by the differential equation

$$\frac{dx}{dt} = x(1 - x).$$

(i) Verify that $x = \frac{1}{1 + e^{-t}}$ satisfies this differential equation, including the initial condition. **[6]**

(ii) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. **[3]**

An alternative model for this situation is given by the differential equation

$$\frac{dx}{dt} = x^2(1 - x),$$

with $x = 0.5$ when $t = 0$ as before.

(iii) Find constants A, B and C such that $\frac{1}{x^2(1 - x)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{1 - x}$. **[4]**

(iv) Hence show that $t = 2 + \ln\left(\frac{x}{1 - x}\right) - \frac{1}{x}$. **[5]**

(v) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. **[2]**

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ADVANCED GCE

MATHEMATICS (MEI) **4754A**

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Turn over

Section A (36 marks)

1 Express $\dfrac{x}{x^2-1}+\dfrac{2}{x+1}$ as a single fraction, simplifying your answer. **[3]**

2 Fig. 2 shows the curve $y=\sqrt{1+x^2}$.

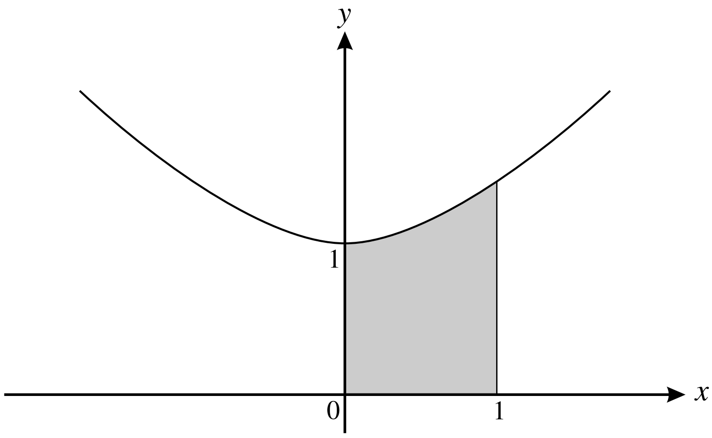

Fig. 2

(i) The following table gives some values of x and y.

x	0	0.25	0.5	0.75	1
y	1	1.0308		1.25	1.4142

Find the missing value of y, giving your answer correct to 4 decimal places.

Hence show that, using the trapezium rule with four strips, the shaded area is approximately 1.151 square units. **[3]**

(ii) Jenny uses a trapezium rule with 8 strips, and obtains a value of 1.158 square units. Explain why she must have made a mistake. **[2]**

(iii) The shaded area is rotated through 360° about the x-axis. Find the exact volume of the solid of revolution formed. **[3]**

3 The parametric equations of a curve are

$$x=\cos 2\theta, \quad y=\sin\theta\cos\theta \quad \text{for } 0 \leqslant \theta < \pi.$$

Show that the cartesian equation of the curve is $x^2+4y^2=1$.

Sketch the curve. **[5]**

4 Find the first three terms in the binomial expansion of $\sqrt{4+x}$ in ascending powers of x.

State the set of values of x for which the expansion is valid. **[5]**

5 **(i)** Express $\dfrac{3}{(y-2)(y+1)}$ in partial fractions. **[3]**

(ii) Hence, given that x and y satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2(y-2)(y+1),$$

show that $\dfrac{y-2}{y+1} = A\mathrm{e}^{x^3}$, where A is a constant. **[5]**

6 Solve the equation $\tan(\theta + 45°) = 1 - 2\tan\theta$, for $0° \leqslant \theta \leqslant 90°$. **[7]**

Section B (36 marks)

7 A straight pipeline AB passes through a mountain. With respect to axes Oxyz, with Ox due East, Oy due North and Oz vertically upwards, A has coordinates (–200, 100, 0) and B has coordinates (100, 200, 100), where units are metres.

(i) Verify that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ and find the length of the pipeline. **[3]**

(ii) Write down a vector equation of the line AB, and calculate the angle it makes with the vertical. **[6]**

A thin flat layer of hard rock runs through the mountain. The equation of the plane containing this layer is $x + 2y + 3z = 320$.

(iii) Find the coordinates of the point where the pipeline meets the layer of rock. **[4]**

(iv) By calculating the angle between the line AB and the normal to the plane of the layer, find the angle at which the pipeline cuts through the layer. **[5]**

[Question 8 is printed overleaf.]

8 Part of the track of a roller-coaster is modelled by a curve with the parametric equations

$$x = 2\theta - \sin\theta, \quad y = 4\cos\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

This is shown in Fig. 8. B is a minimum point, and BC is vertical.

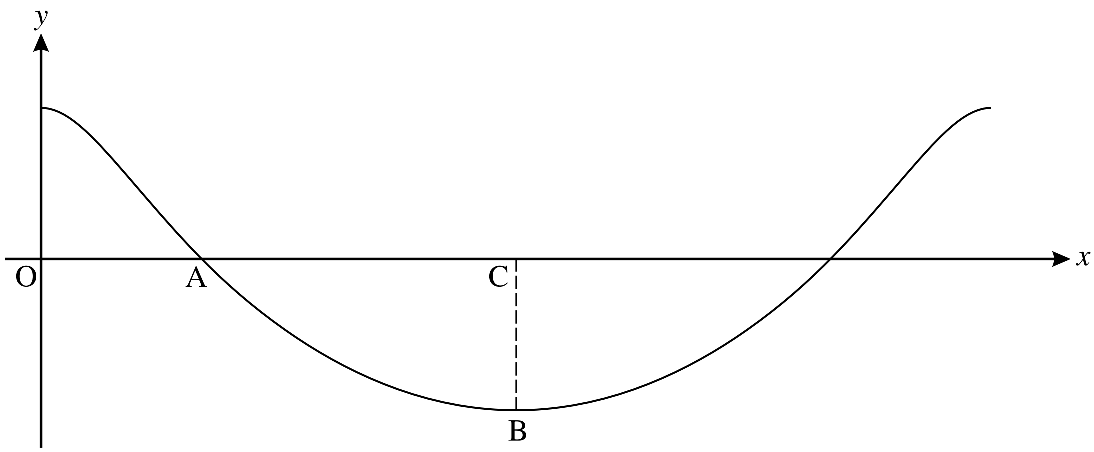

Fig. 8

(i) Find the values of the parameter at A and B.

Hence show that the ratio of the lengths OA and AC is $(\pi - 1) : (\pi + 1)$. **[5]**

(ii) Find $\dfrac{dy}{dx}$ in terms of θ. Find the gradient of the track at A. **[4]**

(iii) Show that, when the gradient of the track is 1, θ satisfies the equation

$$\cos\theta - 4\sin\theta = 2.$$ **[2]**

(iv) Express $\cos\theta - 4\sin\theta$ in the form $R\cos(\theta + \alpha)$.

Hence solve the equation $\cos\theta - 4\sin\theta = 2$ for $0 \leqslant \theta \leqslant 2\pi$. **[7]**

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ADVANCED GCE

MATHEMATICS (MEI) **4754A**

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Friday 14 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Turn over

Section A (36 marks)

1 **(i)** Use the trapezium rule with four strips to estimate $\int_{-2}^{2} \sqrt{1+e^x}\,dx$, showing your working. **[4]**

Fig. 1 shows a sketch of $y = \sqrt{1+e^x}$.

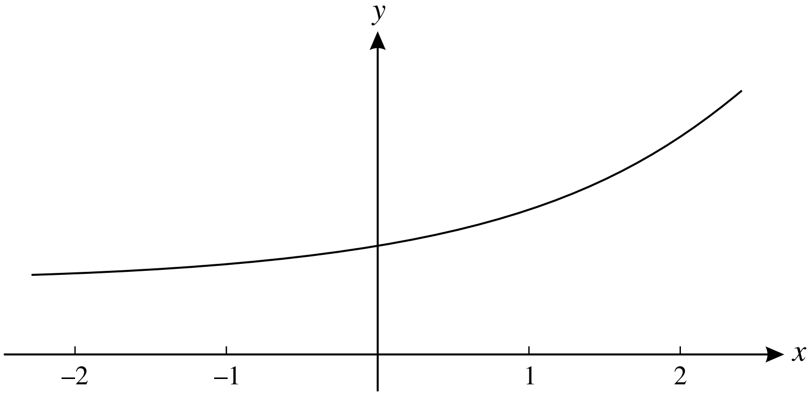

Fig. 1

(ii) Suppose that the trapezium rule is used with more strips than in part **(i)** to estimate $\int_{-2}^{2} \sqrt{1+e^x}\,dx$. State, with a reason but no further calculation, whether this would give a larger or smaller estimate. **[2]**

2 A curve is defined parametrically by the equations

$$x = \frac{1}{1+t}, \qquad y = \frac{1-t}{1+2t}.$$

Find t in terms of x. Hence find the cartesian equation of the curve, giving your answer as simply as possible. **[5]**

3 Find the first three terms in the binomial expansion of $\frac{1}{(3-2x)^3}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[7]**

4 The points A, B and C have coordinates (2, 0, −1), (4, 3, −6) and (9, 3, −4) respectively.

(i) Show that AB is perpendicular to BC. **[4]**

(ii) Find the area of triangle ABC. **[3]**

5 Show that $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$. **[3]**

6 **(i)** Find the point of intersection of the line $\mathbf{r} = \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ and the plane $2x - 3y + z = 11$. **[4]**

(ii) Find the acute angle between the line and the normal to the plane. **[4]**

Section B (36 marks)

7 A particle is moving vertically downwards in a liquid. Initially its velocity is zero, and after t seconds it is $v\,\text{m s}^{-1}$. Its terminal (long-term) velocity is $5\,\text{m s}^{-1}$.

A model of the particle's motion is proposed. In this model, $v = 5\left(1 - \mathrm{e}^{-2t}\right)$.

(i) Show that this equation is consistent with the initial and terminal velocities. Calculate the velocity after 0.5 seconds as given by this model. **[3]**

(ii) Verify that v satisfies the differential equation $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 10 - 2v$. **[3]**

In a second model, v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 10 - 0.4v^2.$$

As before, when $t = 0$, $v = 0$.

(iii) Show that this differential equation may be written as

$$\frac{10}{(5 - v)(5 + v)}\frac{\mathrm{d}v}{\mathrm{d}t} = 4.$$

Using partial fractions, solve this differential equation to show that

$$t = \tfrac{1}{4}\ln\left(\frac{5 + v}{5 - v}\right).$$ **[8]**

This can be re-arranged to give $v = \dfrac{5\left(1 - \mathrm{e}^{-4t}\right)}{1 + \mathrm{e}^{-4t}}$. [You are **not** required to show this result.]

(iv) Verify that this model also gives a terminal velocity of $5\,\text{m s}^{-1}$.

Calculate the velocity after 0.5 seconds as given by this model. **[3]**

The velocity of the particle after 0.5 seconds is measured as $3\,\text{m s}^{-1}$.

(v) Which of the two models fits the data better? **[1]**

8 Fig. 8 shows a searchlight, mounted at a point A, 5 metres above level ground. Its beam is in the shape of a cone with axis AC, where C is on the ground. AC is angled at α to the vertical. The beam produces an oval-shaped area of light on the ground, of length DE. The width of the oval at C is GF. Angles DAC, EAC, FAC and GAC are all β.

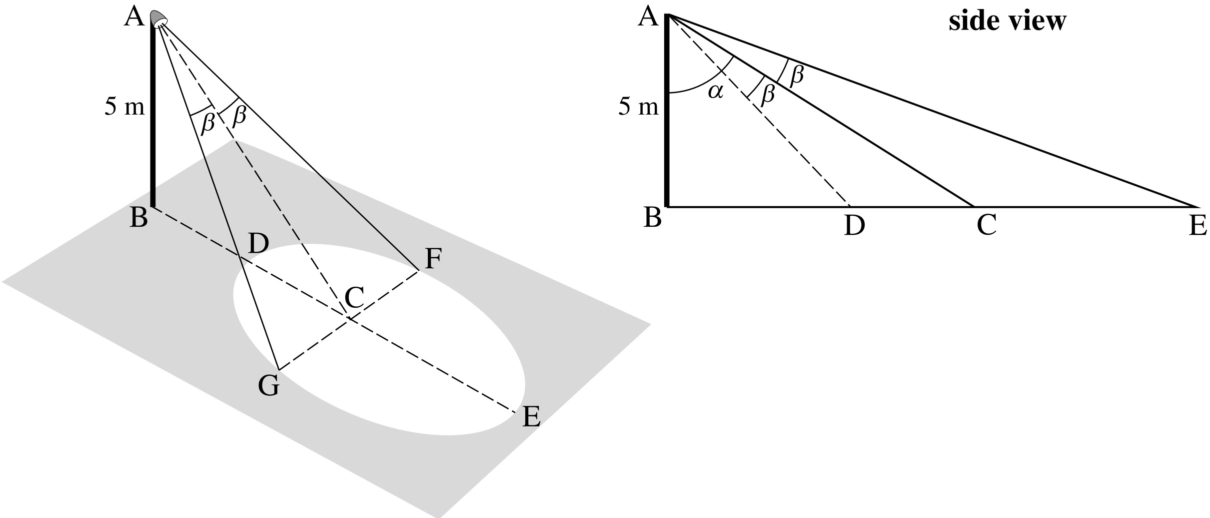

Fig. 8

In the following, all lengths are in metres.

(i) Find AC in terms of α, and hence show that GF = $10 \sec \alpha \tan \beta$. **[3]**

(ii) Show that CE = $5\big(\tan(\alpha + \beta) - \tan \alpha\big)$.

Hence show that CE = $\dfrac{5 \tan \beta \sec^2 \alpha}{1 - \tan \alpha \tan \beta}$. **[5]**

Similarly, it can be shown that CD = $\dfrac{5 \tan \beta \sec^2 \alpha}{1 + \tan \alpha \tan \beta}$. [You are **not** required to derive this result.]

You are now given that $\alpha = 45^\circ$ and that $\tan \beta = t$.

(iii) Find CE and CD in terms of t. Hence show that DE = $\dfrac{20t}{1 - t^2}$. **[5]**

(iv) Show that GF = $10\sqrt{2}t$. **[2]**

For a certain value of β, DE = 2GF.

(v) Show that $t^2 = 1 - \dfrac{1}{\sqrt{2}}$.

Hence find this value of β. **[3]**

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ADVANCED GCE

MATHEMATICS (MEI) **4754A**

Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Express $\dfrac{1}{(2x+1)(x^2+1)}$ in partial fractions. **[5]**

2 Find the first three terms in the binomial expansion of $\sqrt[3]{1+3x}$ in ascending powers of x. State the set of values of x for which the expansion is valid. **[5]**

3 Express $2\sin\theta - 3\cos\theta$ in the form $R\sin(\theta-\alpha)$, where R and α are constants to be determined, and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the greatest and least possible values of $1 + 2\sin\theta - 3\cos\theta$. **[6]**

4 A curve has parametric equations

$$x = 2\sin\theta, \quad y = \cos 2\theta.$$

(i) Find the exact coordinates and the gradient of the curve at the point with parameter $\theta = \frac{1}{3}\pi$. **[5]**

(ii) Find y in terms of x. **[2]**

5 Solve the equation $\operatorname{cosec}^2\theta = 1 + 2\cot\theta$, for $-180° \leqslant \theta \leqslant 180°$. **[6]**

6 Fig. 6 shows the region enclosed by part of the curve $y = 2x^2$, the straight line $x + y = 3$, and the y-axis. The curve and the straight line meet at P (1, 2).

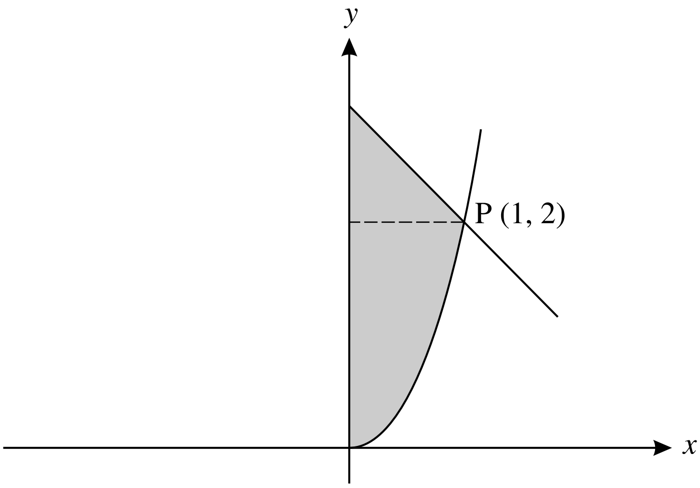

Fig. 6

The shaded region is rotated through 360° about the y-axis. Find, in terms of π, the volume of the solid of revolution formed. **[7]**

[You may use the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.]

Section B (36 marks)

7 A piece of cloth ABDC is attached to the tops of vertical poles AE, BF, DG and CH, where E, F, G and H are at ground level (see Fig. 7). Coordinates are as shown, with lengths in metres. The length of pole DG is k metres.

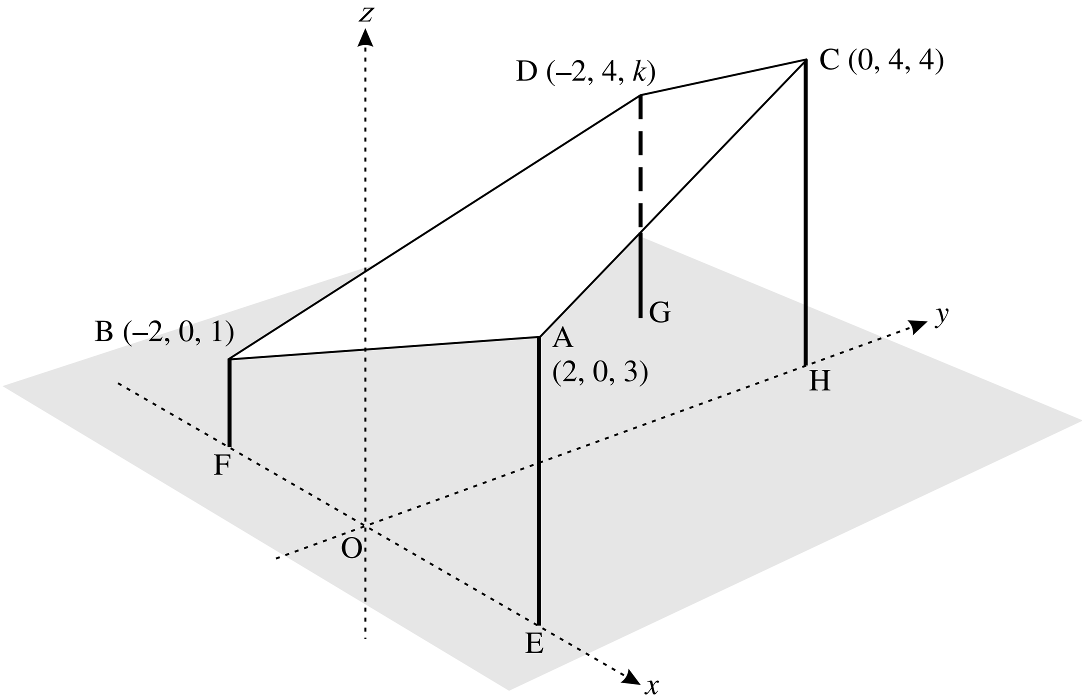

Fig. 7

(i) Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Hence calculate the angle BAC. **[6]**

(ii) Verify that the equation of the plane ABC is $x + y - 2z + d = 0$, where d is a constant to be determined.

Calculate the acute angle the plane makes with the horizontal plane. **[7]**

(iii) Given that A, B, D and C are coplanar, show that $k = 3$.

Hence show that ABDC is a trapezium, and find the ratio of CD to AB. **[5]**

[Question 8 is printed overleaf.]

8 Water is leaking from a container. After t seconds, the depth of water in the container is x cm, and the volume of water is V cm^3, where $V = \frac{1}{3}x^3$. The rate at which water is lost is proportional to x, so that $\frac{dV}{dt} = -kx$, where k is a constant.

(i) Show that $x\frac{dx}{dt} = -k$. **[3]**

Initially, the depth of water in the container is 10 cm.

(ii) Show by integration that $x = \sqrt{100 - 2kt}$. **[4]**

(iii) Given that the container empties after 50 seconds, find k. **[2]**

Once the container is empty, water is poured into it at a constant rate of 1 cm^3 per second. The container continues to lose water as before.

(iv) Show that, t seconds after starting to pour the water in, $\frac{dx}{dt} = \frac{1-x}{x^2}$. **[2]**

(v) Show that $\frac{1}{1-x} - x - 1 = \frac{x^2}{1-x}$.

Hence solve the differential equation in part **(iv)** to show that

$$t = \ln\left(\frac{1}{1-x}\right) - \frac{1}{2}x^2 - x.$$ **[6]**

(vi) Show that the depth cannot reach 1 cm. **[1]**

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Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

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- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Express $\dfrac{x+1}{x^2(2x-1)}$ in partial fractions. **[5]**

2 Solve, correct to 2 decimal places, the equation $\cot 2\theta = 3$ for $0° \leqslant \theta \leqslant 180°$. **[4]**

3 Express $3\sin x + 2\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence find, correct to 2 decimal places, the coordinates of the maximum point on the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 3\sin x + 2\cos x, \quad 0 \leqslant x \leqslant \pi.$$ **[7]**

4 **(i)** Complete the table of values for the curve $y = \sqrt{\cos x}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y		0.9612	0.8409		

Hence use the trapezium rule with strip width $h = \frac{\pi}{8}$ to estimate the value of the integral $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\,\mathrm{d}x$, giving your answer to 3 decimal places. **[3]**

Fig. 4 shows the curve $y = \sqrt{\cos x}$ for $0 \leqslant x \leqslant \frac{\pi}{2}$.

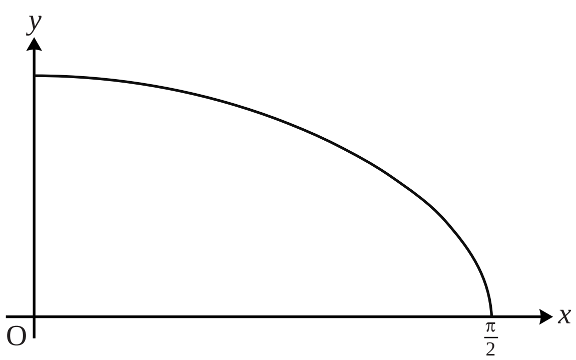

Fig. 4

(ii) State, with a reason, whether the trapezium rule with a strip width of $\frac{\pi}{16}$ would give a larger or smaller estimate of the integral. **[1]**

5 Verify that the vector $2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ is perpendicular to the plane through the points A(2, 0, 1), B(1, 2, 2) and C(0, –4, 1). Hence find the cartesian equation of the plane. **[5]**

6 Given the binomial expansion $(1+qx)^p = 1 - x + 2x^2 + \ldots$, find the values of p and q. Hence state the set of values of x for which the expansion is valid. **[6]**

7 Show that the straight lines with equations $\mathbf{r} = \begin{pmatrix}4\\2\\4\end{pmatrix} + \lambda\begin{pmatrix}3\\0\\1\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}-1\\4\\9\end{pmatrix} + \mu\begin{pmatrix}-1\\1\\3\end{pmatrix}$ meet.

Find their point of intersection. **[5]**

Section B (36 marks)

8 Fig. 8 shows a cross-section of a car headlight whose inside reflective surface is modelled, in suitable units, by the curve

$$x = 2t^2,\ y = 4t, \quad -\sqrt{2} \leqslant t \leqslant \sqrt{2}\ .$$

P$(2t^2, 4t)$ is a point on the curve with parameter t. TS is the tangent to the curve at P, and PR is the line through P parallel to the x-axis. Q is the point (2, 0). The angles that PS and QP make with the positive x-direction are θ and ϕ respectively.

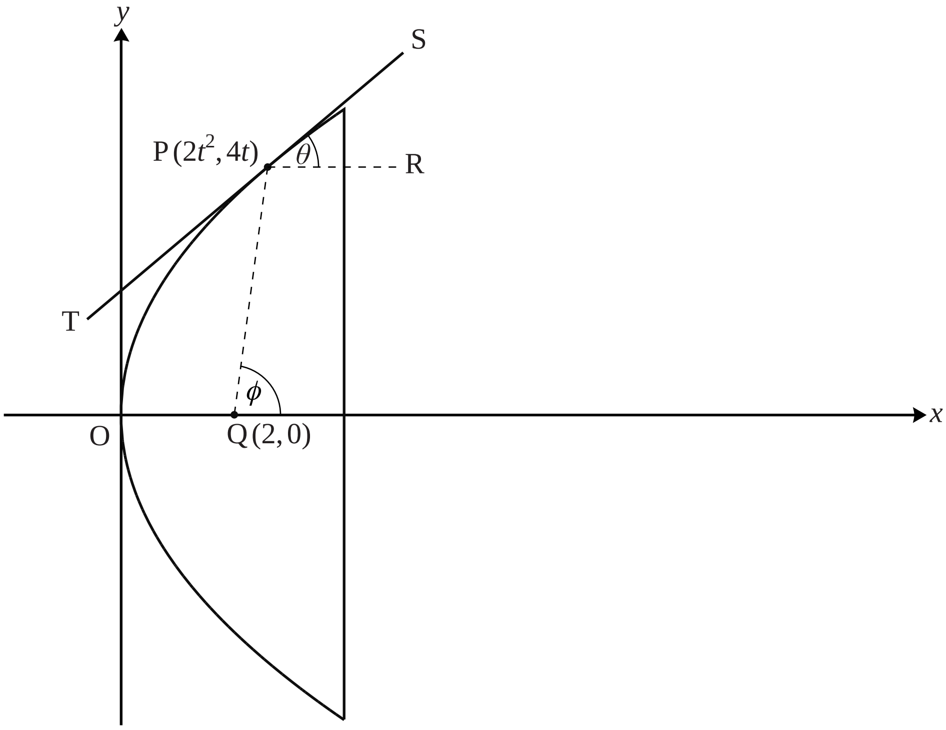

Fig. 8

(i) By considering the gradient of the tangent TS, show that $\tan\theta = \dfrac{1}{t}$. **[3]**

(ii) Find the gradient of the line QP in terms of t. Hence show that $\phi = 2\theta$, and that angle TPQ is equal to θ. **[8]**

[The above result shows that if a lamp bulb is placed at Q, then the light from the bulb is reflected to produce a parallel beam of light.]

The inside surface of the headlight has the shape produced by rotating the curve about the x-axis.

(iii) Show that the curve has cartesian equation $y^2 = 8x$. Hence find the volume of revolution of the curve, giving your answer as a multiple of π. **[7]**

9

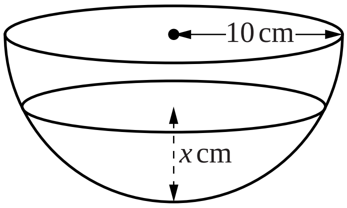

Fig. 9

Fig. 9 shows a hemispherical bowl, of radius 10 cm, filled with water to a depth of x cm. It can be shown that the volume of water, V cm^3, is given by

$$V = \pi(10x^2 - \tfrac{1}{3}x^3).$$

Water is poured into a leaking hemispherical bowl of radius 10 cm. Initially, the bowl is empty. After t seconds, the volume of water is changing at a rate, in cm^3 s^{-1}, given by the equation

$$\frac{dV}{dt} = k(20 - x),$$

where k is a constant.

(i) Find $\frac{dV}{dx}$, and hence show that $\pi x \frac{dx}{dt} = k$. **[4]**

(ii) Solve this differential equation, and hence show that the bowl fills completely after T seconds, where $T = \frac{50\pi}{k}$. **[5]**

Once the bowl is full, the supply of water to the bowl is switched off, and water then leaks out at a rate of kx cm^3 s^{-1}.

(iii) Show that, t seconds later, $\pi(20 - x)\frac{dx}{dt} = -k$. **[3]**

(iv) Solve this differential equation.

Hence show that the bowl empties in $3T$ seconds. **[6]**

OCR
RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754 Applications of Advanced Mathematics (C4)

INSTRUCTIONS

4715680612

The examination is in two parts:

Paper A (1 hour 30 minutes)
Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B **is not issued** until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding paper A script. For Paper A only the candidates' printed answer books, in the same order as the attendance register, should be sent for marking; the question paper should be retained in the centre or recycled. For Paper B only the question papers, on which the candidates have written their answers, should be sent for marking; the insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

<u>For Paper B (Comprehension) only.</u>

A standard English dictionary is allowed for the comprehension.
(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the Invigilator's desk at all times during the morning of Thursday 14 June 2012.

OCR RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1 Solve the equation $\frac{4x}{x+1} - \frac{3}{2x+1} = 1$. **[5]**

2 Find the first four terms in the binomial expansion of $\sqrt{1+2x}$. State the set of values of x for which the expansion is valid. **[5]**

3 The total value of the sales made by a new company in the first t years of its existence is denoted by £V. A model is proposed in which the rate of increase of V is proportional to the square root of V. The constant of proportionality is k.

(i) Express the model as a differential equation.

Verify by differentiation that $V = (\frac{1}{2}kt + c)^2$, where c is an arbitrary constant, satisfies this differential equation. **[4]**

(ii) The value of the company's sales in its first year is £10 000, and the total value of the sales in the first two years is £40 000. Find V in terms of t. **[4]**

4 Prove that $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \operatorname{cosec}^2\theta$. **[4]**

5 Given the equation $\sin(x + 45°) = 2\cos x$, show that $\sin x + \cos x = 2\sqrt{2}\cos x$.

Hence solve, correct to 2 decimal places, the equation for $0° \leqslant x \leqslant 360°$. **[6]**

6 Solve the differential equation $\frac{dy}{dx} = \frac{y}{x(x+1)}$, given that when $x=1$, $y=1$. Your answer should express y explicitly in terms of x. **[8]**

Section B (36 marks)

7 Fig. 7a shows the curve with the parametric equations

$$x = 2\cos\theta, \quad y = \sin 2\theta, \quad -\tfrac{\pi}{2} \leqslant \theta \leqslant \tfrac{\pi}{2}.$$

The curve meets the x-axis at O and P. Q and R are turning points on the curve. The scales on the axes are the same.

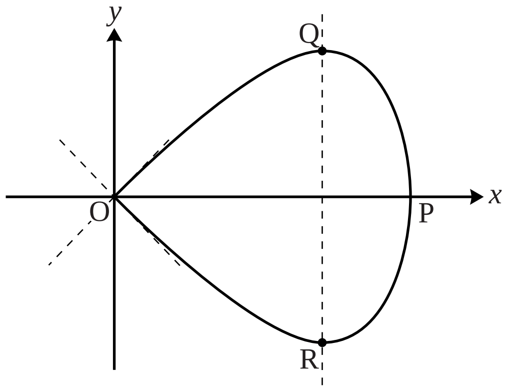

Fig. 7a

(i) State, with their coordinates, the points on the curve for which $\theta = -\frac{\pi}{2}$, $\theta = 0$ and $\theta = \frac{\pi}{2}$. **[3]**

(ii) Find $\frac{dy}{dx}$ in terms of θ. Hence find the gradient of the curve when $\theta = \frac{\pi}{2}$, and verify that the two tangents to the curve at the origin meet at right angles. **[5]**

(iii) Find the exact coordinates of the turning point Q. **[3]**

When the curve is rotated about the x-axis, it forms a paperweight shape, as shown in Fig. 7b.

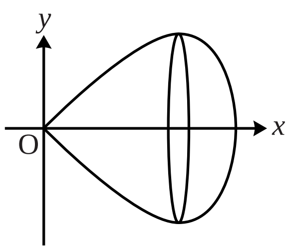

Fig. 7b

(iv) Express $\sin^2\theta$ in terms of x. Hence show that the cartesian equation of the curve is $y^2 = x^2(1 - \frac{1}{4}x^2)$. **[4]**

(v) Find the volume of the paperweight shape. **[4]**

8 With respect to cartesian coordinates Oxyz, a laser beam ABC is fired from the point A(1, 2, 4), and is reflected at point B off the plane with equation $x + 2y - 3z = 0$, as shown in Fig. 8. A′ is the point (2, 4, 1), and M is the midpoint of AA′.

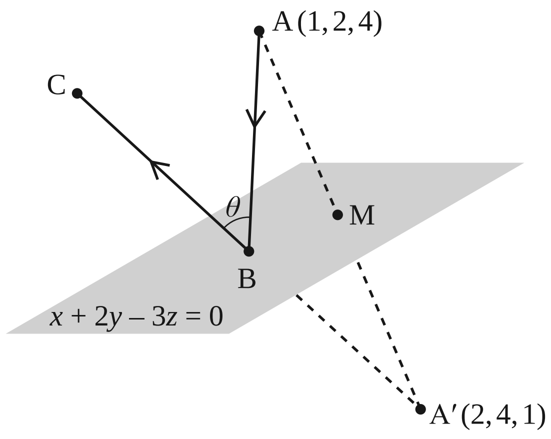

Fig. 8

(i) Show that AA′ is perpendicular to the plane $x + 2y - 3z = 0$, and that M lies in the plane. **[4]**

The vector equation of the line AB is $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

(ii) Find the coordinates of B, and a vector equation of the line A′B. **[6]**

(iii) Given that A′BC is a straight line, find the angle θ. **[4]**

(iv) Find the coordinates of the point where BC crosses the Oxz plane (the plane containing the x- and z-axes). **[3]**

OCR RECOGNISING ACHIEVEMENT

Friday 18 January 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
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- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
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- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1 Solve the equation $\dfrac{2x}{x+1} - \dfrac{1}{x-1} = 1$. **[4]**

2 Find the first four terms of the binomial expansion of $\sqrt[3]{1-2x}$. State the set of values of x for which the expansion is valid. **[6]**

3 The parametric equations of a curve are

$$x = \sin\theta, \quad y = \sin 2\theta, \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

(i) Find the exact value of the gradient of the curve at the point where $\theta = \frac{1}{6}\pi$. **[4]**

(ii) Show that the cartesian equation of the curve is $y^2 = 4x^2 - 4x^4$. **[3]**

4 Fig. 4 shows the curve $y = \sqrt{1 + e^{2x}}$, and the region between the curve, the x-axis, the y-axis and the line $x = 2$.

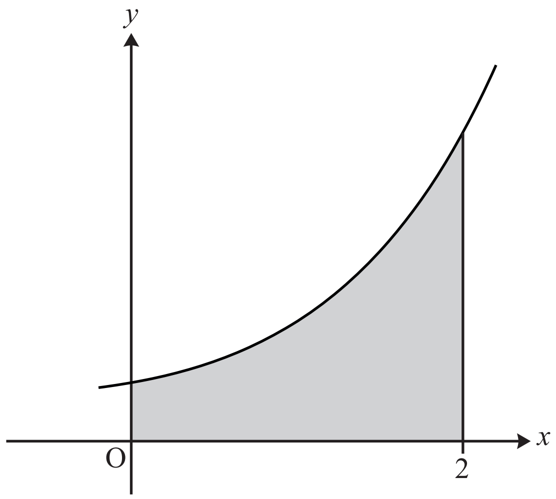

Fig. 4

(a) Find the exact volume of revolution when the shaded region is rotated through 360° about the x-axis. **[4]**

(b) **(i)** Complete the table of values, and use the trapezium rule with 4 strips to estimate the area of the shaded region. **[3]**

x	0	0.5	1	1.5	2
y		1.9283	2.8964	4.5919	

(ii) The trapezium rule for $\int_0^2 \sqrt{1 + e^{2x}}\,dx$ with 8 and 16 strips gives 6.797 and 6.823, although not necessarily in that order. Without doing the calculations, say which result is which, explaining your reasoning. **[1]**

5 Solve the equation $2\sec^2\theta = 5\tan\theta$, for $0 \leqslant \theta \leqslant \pi$. **[6]**

6 In Fig. 6, ABC, ACD and AED are right-angled triangles and BC = 1 unit. Angles CAB and CAD are θ and ϕ respectively.

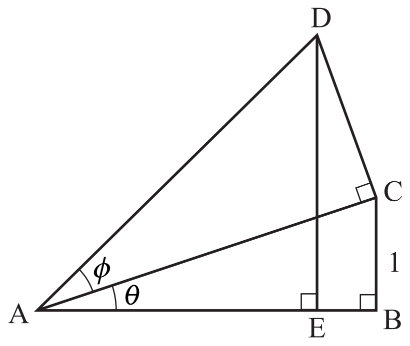

Fig. 6

(i) Find AC and AD in terms of θ and ϕ. **[2]**

(ii) Hence show that $\text{DE} = 1 + \dfrac{\tan\phi}{\tan\theta}$. **[3]**

Section B (36 marks)

7 A tent has vertices ABCDEF with coordinates as shown in Fig. 7. Lengths are in metres. The Oxy plane is horizontal.

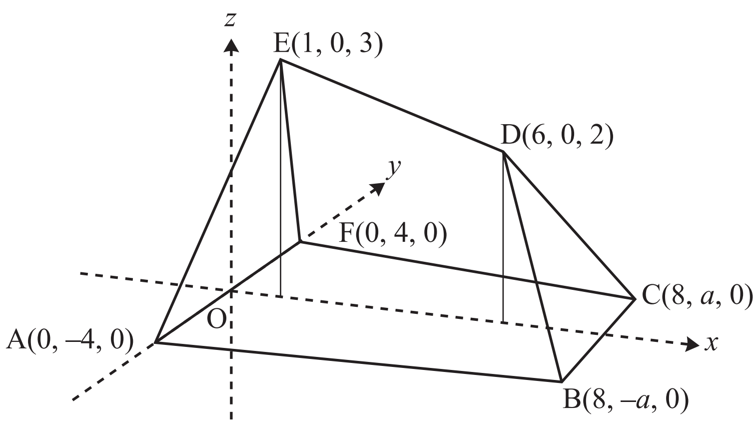

Fig. 7

(i) Find the length of the ridge of the tent DE, and the angle this makes with the horizontal. **[4]**

(ii) Show that the vector $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ is normal to the plane through A, D and E.

Hence find the equation of this plane. Given that B lies in this plane, find a. **[7]**

(iii) Verify that the equation of the plane BCD is $x + z = 8$.

Hence find the acute angle between the planes ABDE and BCD. **[6]**

8 The growth of a tree is modelled by the differential equation

$$10\frac{\mathrm{d}h}{\mathrm{d}t} = 20 - h,$$

where h is its height in metres and the time t is in years. It is assumed that the tree is grown from seed, so that $h = 0$ when $t = 0$.

(i) Write down the value of h for which $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$, and interpret this in terms of the growth of the tree. **[1]**

(ii) Verify that $h = 20\left(1 - \mathrm{e}^{-0.1t}\right)$ satisfies this differential equation and its initial condition. **[5]**

The alternative differential equation

$$200\frac{\mathrm{d}h}{\mathrm{d}t} = 400 - h^2$$

is proposed to model the growth of the tree. As before, $h = 0$ when $t = 0$.

(iii) Using partial fractions, show by integration that the solution to the alternative differential equation is

$$h = \frac{20\left(1 - \mathrm{e}^{-0.2t}\right)}{1 + \mathrm{e}^{-0.2t}}.$$

[9]

(iv) What does this solution indicate about the long-term height of the tree? **[1]**

(v) After a year, the tree has grown to a height of 2 m. Which model fits this information better? **[3]**

OCR RECOGNISING ACHIEVEMENT

Thursday 13 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1 **(i)** Express $\dfrac{x}{(1+x)(1-2x)}$ in partial fractions. **[3]**

(ii) Hence use binomial expansions to show that $\dfrac{x}{(1+x)(1-2x)} = ax + bx^2 + \ldots$, where a and b are constants to be determined.

State the set of values of x for which the expansion is valid. **[5]**

2 Show that the equation $\operatorname{cosec} x + 5\cot x = 3\sin x$ may be rearranged as

$$3\cos^2 x + 5\cos x - 2 = 0.$$

Hence solve the equation for $0° \leqslant x \leqslant 360°$, giving your answers to 1 decimal place. **[7]**

3 Using appropriate right-angled triangles, show that $\tan 45° = 1$ and $\tan 30° = \dfrac{1}{\sqrt{3}}$.

Hence show that $\tan 75° = 2 + \sqrt{3}$. **[7]**

4 **(i)** Find a vector equation of the line l joining the points $(0, 1, 3)$ and $(-2, 2, 5)$. **[2]**

(ii) Find the point of intersection of the line l with the plane $x + 3y + 2z = 4$. **[3]**

(iii) Find the acute angle between the line l and the normal to the plane. **[3]**

5 The points A, B and C have coordinates $\mathrm{A}(3, 2, -1)$, $\mathrm{B}(-1, 1, 2)$ and $\mathrm{C}(10, 5, -5)$, relative to the origin O. Show that $\overrightarrow{\mathrm{OC}}$ can be written in the form $\lambda\overrightarrow{\mathrm{OA}} + \mu\overrightarrow{\mathrm{OB}}$, where λ and μ are to be determined.

What can you deduce about the points O, A, B and C from the fact that $\overrightarrow{\mathrm{OC}}$ can be expressed as a combination of $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{OB}}$? **[6]**

Section B (36 marks)

6 The motion of a particle is modelled by the differential equation

$$v\frac{\mathrm{d}v}{\mathrm{d}x} + 4x = 0,$$

where x is its displacement from a fixed point, and v is its velocity.

Initially $x = 1$ and $v = 4$.

(i) Solve the differential equation to show that $v^2 = 20 - 4x^2$. **[4]**

Now consider motion for which $x = \cos 2t + 2\sin 2t$, where x is the displacement from a fixed point at time t.

(ii) Verify that, when $t = 0$, $x = 1$. Use the fact that $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ to verify that when $t = 0$, $v = 4$. **[4]**

(iii) Express x in the form $R\cos(2t - \alpha)$, where R and α are constants to be determined, and obtain the corresponding expression for v. Hence or otherwise verify that, for this motion too, $v^2 = 20 - 4x^2$. **[7]**

(iv) Use your answers to part **(iii)** to find the maximum value of x, and the earliest time at which x reaches this maximum value. **[3]**

7 Fig. 7 shows the curve BC defined by the parametric equations

$$x = 5\ln u,\ y = u + \frac{1}{u},\quad 1 \leqslant u \leqslant 10.$$

The point A lies on the x-axis and AC is parallel to the y-axis. The tangent to the curve at C makes an angle θ with AC, as shown.

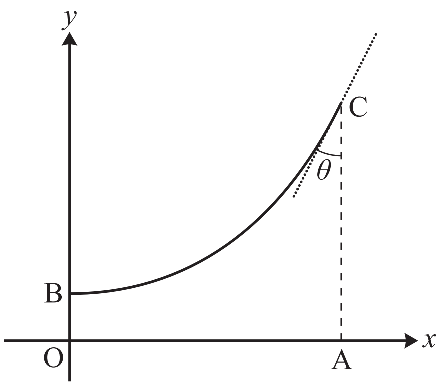

Fig. 7

(i) Find the lengths OA, OB and AC. **[5]**

(ii) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of u. Hence find the angle θ. **[6]**

(iii) Show that the cartesian equation of the curve is $y = \mathrm{e}^{\frac{1}{5}x} + \mathrm{e}^{-\frac{1}{5}x}$. **[2]**

An object is formed by rotating the region OACB through 360° about Ox.

(iv) Find the volume of the object. **[5]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.